ECE 443/643 Final

December 15, 2011

1. Let $x(t) = (\text{sinc}(t))^2$.
   (a) *(5)* Find and sketch $X(f)$.
   (b) *(5)* Find the energy spectral density of x.
   (c) *(5)* Find the energy of x.
   (d) Let $y(t)$ be the output of an ideal low pass filter (i.e. $Y(f) = X(f)$ in the pass band, and $Y(f) = 0$ otherwise). The cutoff frequency for the filter is such that the energy of y is exactly half that of x.
      i. *(10)* What is the cutoff frequency?
      ii. *(5)* Find and sketch $Y(f)$.
      iii. *(10)* Find $y(t)$.

2. *(5)* Find the Hilbert transform of $\delta(t)$.

3. Consider a superhet with a broken RF filter (the image frequencies leak through to the IF stage). It uses high side injection (i.e. $f_{LO} > f_c$), is designed for receiving frequencies $50 < f_c < 54$ MHz, and has $f_{IF} = 7$ MHz.
   (a) *(5)* What is the range of f_{LO}?
   (b) *(10)* What is the range(s) of frequencies that can be mixed down to IF, given the range of f_{LO}?

4. Consider an FM signal $x(t)$ with message $m(t) = \cos(2\pi f_m t)$, an amplitude of 1, carrier frequency f_c =1 MHz, and peak frequency deviation f_Δ =10 kHz.
   (a) *(5)* Write an expression for the instantaneous frequency f_i.
   (b) *(5)* Write an expression for $x(t)$.
   (c) *(5)* Estimate the bandwidth of $x(t)$.
   (d) *(10)* Draw a block diagram for a system that converts $x(t)$ to an FM signal with f_c = 40MHz and f_Δ = 500kHz. You may only use 2×, 3×, 4×, and 5× frequency multipliers.

5. For each of the following, answer with a few sentences.
   (a) *(5)* What is an advantage of FM over AM? A disadvantage?
   (b) *(5)* Describe at least 2 reasons to shift a signal in frequency for transmission.

6. *(10)* A wide sense stationary signal $x(t)$ is input to a Hilbert transformer to give output $\hat{x}(t)$. Find the PSD $G_{\hat{x}}(f)$ in terms of the input PSD $G_x(f)$.